

A GENERALIZED THEORY OF THERMOELASTICITY FOR PRESTRESSED BODIES WITH MICROSTRUCTURE

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Abstract—A generalized theory is presented for a thermoelastic dipolar body which has previously received a large deformation and is at nonuniform temperature. A generalized linear theory of dipolar thermoelasticity with initial stress and initial heat flux has been derived.

NOMENCLATURE

$E_{AB}, E_{A:B}, E_{A:BK}$	the strain tensors
f_k	external body force per unit mass
f_{iK}	dipolar body force per unit mass
Q_K	the heat flux across surfaces that were originally coordinate planes perpendicular to the X_K axes, measured per unit area of these planes, per unit time
s	the heat supply per unit mass and time
S_{KIB}, S_{AB}	the generalized stress tensors
T	absolute temperature
T_{Ki}	first Piola-Kirchhoff stress tensor ($T_{Ki} \neq T_{iK}$)
Y_{XL}	the inertia coefficients
Ψ	Gibbs free energy per unit of initial volume
ρ_0	initial mass density
δ_{AB}	Kronecker delta
η	the entropy per unit mass.

1. INTRODUCTION

During the past two decades, many papers have been devoted to the generalized theory of thermoelasticity, which avoid the paradox of propagation of heat with infinite speed. Some of the papers worth mentioning are by Lord and Shulman (1967), Kaliski and Nowacki (1970), Green and Lindsay (1972), Boschi and Iesan (1973), Dost and Tabarrok (1978), Sherief and Dhaliwal (1980), Smirnov (1981), Lebon (1982), and Chandrasekharaiah (1986). This paradox is produced by the diffusion type equation governing the temperature field in the classical theories.

The purpose of the present paper is to establish a generalized theory for a thermoelastic dipolar body which has been prestressed. The coupled theory for a dipolar prestressed thermoelastic body has been obtained by Iesan (1981).

In Section 2, we modify the nonlinear constitutive equations of dipolar thermoelastic bodies developed by Green and Rivlin (1964), which provide a foundation for our later work. In Section 3, we derive the generalized thermoelastic equations for a dipolar body which has previously received large deformation and is at nonuniform temperature. These results are then used, in Section 4, to derive the basic equations for a special case concerning small thermoelastic deformations in a dipolar body which is under initial stress and initial heat flux.

2. NONLINEAR FORMULAE

Consider a thermoelastic dipolar body, whose configuration changes continuously under external mechanical actions and heating, from an original reference state Ω_0 to a deformed state Ω . Let $\partial\Omega_0$ and $\partial\Omega$ be the surfaces of Ω_0 and Ω , respectively. X_K will denote

the position of the particle X in the reference state, and x_k the position in the deformed state, where $x_k = x_k(X_1, X_2, X_3, t)$ and t is time.

In the theory of dipolar thermoelastic bodies developed by Green and Rivlin (1964), the deformation is described by the displacement $x_k = x_k(X_1, X_2, X_3, t)$ and dipolar displacement $x_{kA} = x_{kA}(X_1, X_2, X_3, t)$. Let X_{kA} be the value of x_{kA} in the reference state and $X_{BA} = \delta_{kB}X_{kA}$. We assume that x_k and x_{kA} are sufficiently smooth, and that

$$J \equiv \det(\partial x_i / \partial X_A) > 0 \quad \text{and} \quad \det(x_{kA}) \neq 0.$$

In the material description, the basic nonlinear equations in dipolar thermoelasticity take the form:

$$T_{Kk,K} + \rho_0 f_k = \rho_0 \ddot{x}_k, \quad (1)$$

$$S_{MiK,M} - x_{i,A} S_{AK} + \rho_0 f_{iK} = \rho_0 Y_{MK} \ddot{x}_{iM}, \quad (2)$$

$$\rho_0 T \dot{\eta} = \rho_0 s - Q_{A,A} \quad (3)$$

in $\Omega_0 \times [0, t_0)$, where t_0 is some time instant that may be infinite.

The constitutive variables are E_{AB} , $E_{A:B}$, $E_{A:BK}$, T and $T_{,K}$ with the geometrical equations

$$2E_{AB} = x_{i,A} x_{i,B} - \delta_{AB} \quad (4)$$

$$E_{A:B} = x_{i,B} x_{iA} - X_{BA}, \quad (5)$$

$$E_{A:BK} = x_{i,B} x_{iA,K} - X_{BA,K} \quad (6)$$

and the constitutive equations are

$$P_{AB} = X_{A,i} (T_{Bi} - S_{BC} x_{iC} - T_{KBC} x_{iC,K}) = P_{BA} = \partial \Psi / \partial E_{AB}, \quad (7)$$

$$T_{KAB} = X_{A,i} S_{KiB} = \partial \Psi / \partial E_{BA:K}, \quad (8)$$

$$s_{AB} = \partial \Psi / \partial E_{B:A}, \quad (9)$$

$$\eta = - \frac{1}{\rho_0} \partial \Psi / \partial T, \quad \partial \Psi / \partial T_{,K} = 0. \quad (10)$$

A superposed dot stands for the material time derivative while a comma followed by a subscript denotes partial derivatives with respect to the spatial coordinates, in the deformed configuration if the subscript is a minuscule, in the reference configuration if the subscript is a majuscule; that is, $g_{,A} = \partial g / \partial X_A$ and $g_{,k} = \partial g / \partial x_k$. Einstein summation on repeated indices is also used throughout this paper.

We assume that Ψ , T_{KL} , η , Q_K are written as symmetric functions of E_{KL} in the indices K and L , and E_{KL} is understood to mean $(E_{KL} + E_{LK})/2$ in $\partial \Psi / \partial E_{KL}$.

We assume that the heat flux Q_K satisfies a first order rate equation of the form [see Chandrasekharaiah (1986) and Müller (1967)]:

$$\dot{Q}_K = \mathcal{L}_K(E_{AB}, E_{A:B}, E_{A:BL}, T, T_{,L}, Q_K), \quad (11)$$

with $\partial \dot{Q}_K / \partial Q_L = 0$ if $K \neq L$. Since it is required that Q_K vanishes at equilibrium, i.e. for $T_{,L} = 0$, we must also have that

$$\mathcal{L}_K(E_{AB}, E_{A:B}, E_{A:BL}, T, 0, 0) = 0. \tag{12}$$

3. SMALL DEFORMATION SUPERPOSED ON A LARGE DEFORMATION

In this section, we consider three states of the body, the initial state Ω_0 , the first deformed state Ω , and the final deformed state Ω^* corresponding respectively to temperature T_0 and zero displacement; temperature T and displacement \mathbf{v} ; and temperature T^* and displacement $\mathbf{v} + \mathbf{u}$. Following Iesan (1981), we call Ω the primary state and Ω^* the secondary state. Thermodynamic quantities and forces associated with Ω^* will be denoted with an asterisk. The position coordinates of the particle X_A at time t in Ω^* will be denoted by $y_i = y_i(X_1, X_2, X_3, t)$ and $y_{iA} = y_{iA}(X_1, X_2, X_3, t)$ with

$$J^* = \det(y_{iA}) > 0 \quad \text{and} \quad \det(y_{iA}) \neq 0.$$

We shall define the incremental displacement u_i , incremental dipolar displacement u_{iA} and incremental temperature θ , respectively, as

$$u_i = y_i - x_i, \quad u_{iA} = y_{iA} - x_{iA}, \quad \theta = T^* - T. \tag{13}$$

Here we consider the case in which u_i, u_{iA} and θ are small. Suppose that there exists a real parameter ϵ , small enough for its squares and higher powers to be neglected, such that

$$u_i = \epsilon u'_i, \quad u_{iA} = \epsilon u'_{iA}, \quad \theta = \epsilon \theta', \tag{14}$$

where u'_i, u'_{iA} and θ' are independent of ϵ .

In the secondary state Ω^* , the basic equations are of the same form as those in the primary state Ω . We have the equations of motion

$$T^*_{kk,K} + \rho_0 f^*_k = \rho_0 \ddot{y}_k \tag{15}$$

$$S^*_{MiK,M} - x_{iA} S^*_{AK} + \rho_0 f^*_{iK} = \rho_0 Y^*_{MK} \ddot{x}_{iM}, \tag{16}$$

the energy equation

$$\rho_0 T^* \dot{\eta}^* = \rho_0 s^* - Q^*_{A:A}, \tag{17}$$

the geometrical equations

$$2E^*_{AB} = y_{iA} y_{iB} - \delta_{AB}, \tag{18}$$

$$E^*_{A:B} = y_{iB} y_{iA} - X_{BA}, \tag{19}$$

$$E^*_{A:BK} = y_{iB} y_{iA,K} - X_{BA:K} \tag{20}$$

and the constitutive equations are

$$P^*_{AB} = \frac{\partial X_A}{\partial y_i} (T^*_{Bi} - S^*_{BC} y_{iC} - T^*_{kBC} y_{iC,K}) = P^*_{BA} = \partial \Psi^* / \partial E^*_{AB}, \tag{21}$$

$$T^*_{KAB} = \frac{\partial X_A}{\partial y_i} S^*_{kiB} = \partial \Psi^* / \partial E^*_{BA:K}, \tag{22}$$

$$S^*_{AB} = \partial \Psi^* / \partial E^*_{B:A}, \tag{23}$$

$$\eta^* = -\frac{1}{\rho_0} \partial \Psi^* / \partial T^*, \quad \partial \Psi^* / \partial T^*_{,K} = 0, \tag{24}$$

$$\dot{Q}^*_K(X_K, t) = \mathcal{L}_K(E^*_{AB}, E^*_{A:B}, E^*_{A:BL}, T^*, T^*_{,L}, Q^*_K). \tag{25}$$

Now we shall derive the equations satisfied by u_i, u_{iA} and θ . Keeping (14) in mind, we get the following second order approximation :

$$\begin{aligned} &\Psi(E^*_{AB}, E^*_{A:B}, E^*_{A:BK}, T^*) - \Psi(E_{AB}, E_{A:B}, E_{A:BK}, T) \\ &= a\theta + c_{AB}(E^*_{AB} - E_{AB}) + c_{A:B}(E^*_{A:B} - E_{A:B}) + c_{A:BK}(E^*_{A:BK} - E_{A:BK}) \\ &\quad - \frac{1}{2}d\theta^2 - B_{AB}\theta(E^*_{AB} - E_{AB}) - B_{A:B}\theta(E^*_{A:B} - E_{A:B}) - B_{A:BK}\theta(E^*_{A:BK} - E_{A:BK}) \\ &\quad + \frac{1}{2}D_{ABCD}(E^*_{AB} - E_{AB})(E^*_{C:D} - E_{C:D}) + \frac{1}{2}D_{ABCDK}(E^*_{AB} - E_{AB})(E^*_{C:DK} - E_{C:DK}) \\ &\quad + \frac{1}{2}G_{ABCDK}(E^*_{A:B} - E_{A:B})(E^*_{C:DK} - E_{C:DK}) + \frac{1}{2}C_{ABCD}(E^*_{AB} - E_{AB})(E^*_{CD} - E_{CD}) \\ &\quad + \frac{1}{2}C'_{ABCD}(E^*_{A:B} - E_{A:B})(E^*_{C:D} - E_{C:D}) \\ &\quad + \frac{1}{2}C_{ABKLMN}(E^*_{A:BK} - E_{A:BK})(E^*_{L:MN} - E_{L:MN}), \end{aligned} \tag{26}$$

where $B_{AB}, C_{ABCD}, C'_{ABCD}$ and C_{ABKLMN} have certain symmetries in their indices, and that

$$a = \partial \Psi / \partial T = -\rho_0 \eta, \quad c_{AB} = \partial \Psi / \partial E_{AB} = P_{AB}, \tag{27}$$

$$c_{A:B} = \partial \Psi / \partial E_{A:B} = S_{BA}, \quad c_{A:BK} = \partial \Psi / \partial E_{A:BK} = T_{KBA}. \tag{28}$$

From eqns (21)–(24), (26)–(28), we find that

$$P^*_{AB} = P_{AB} - B_{AB}\theta + D_{ABCD}(E^*_{C:D} - E_{C:D}) + D_{ABCDK}(E^*_{C:DK} - E_{C:DK}) + C_{ABCD}(E^*_{CD} - E_{CD}), \tag{29}$$

$$S^*_{BA} = S_{BA} - B_{A:B}\theta + D_{ABCD}(E^*_{CD} - E_{CD}) + G_{ABCDK}(E^*_{C:DK} - E_{C:DK}) + C'_{ABCD}(E^*_{C:D} - E_{C:D}), \tag{30}$$

$$\begin{aligned} T^*_{KBA} &= T_{KBA} - B_{A:BK}\theta + D_{CDABK}(E^*_{CD} - E_{CD}) \\ &\quad + G_{CDABK}(E^*_{C:D} - E_{C:D}) + C_{ABKLMN}(E^*_{L:MN} - E_{L:MN}), \end{aligned} \tag{31}$$

$$\rho_0 \eta^* = \rho_0 \eta + d\theta + B_{AB}(E^*_{AB} - E_{AB}) + B_{A:B}(E^*_{A:B} - E_{A:B}) + B_{A:BK}(E^*_{A:BK} - E_{A:BK}). \tag{32}$$

Also, from eqns (11) and (25), we have

$$\begin{aligned} \dot{Q}^*_K - \dot{Q}_K &= -\frac{1}{\tau} \{ (Q^*_K - Q_K) + a_K \theta + k_{KL} \theta_{,L} + h_{KMN}(E^*_{MN} - E_{MN}) \\ &\quad + h'_{KMN}(E^*_{M:N} - E_{M:N}) + h_{KMNL}(E^*_{M:NL} - E_{M:NL}) \}. \end{aligned} \tag{33}$$

From eqn (12), we find that

$$a_K = 0, \quad h_{KMN} = 0, \quad h'_{KMN} = 0, \quad h_{KMNL} = 0, \quad \text{if } T = \text{constant}. \tag{34}$$

From eqns (4)–(6) and (18)–(20), and the relations

$$y_{iA} = x_{iA} + u_{iA}, \quad y_{iA,B} = x_{iA,B} + u_{iA,B}, \tag{35}$$

we find that

$$2E_{AB}^* = 2E_{AB} + x_{i,A}u_{i,B} + x_{i,B}u_{i,A}, \tag{36}$$

$$E_{A:B}^* = E_{A:B} + x_{i,A}u_{i,B} + x_{i,B}u_{i,A}, \tag{37}$$

$$E_{A:BC}^* = E_{A:BC} + x_{i,A,C}u_{i,B} + x_{i,B}u_{i,A,C}, \tag{38}$$

where we have used the fact that u_i s and u_{iB} s are small and hence the terms like $u_{i,A}u_{i,B}$ have been neglected. Let

$$e_{AB} = \frac{1}{2}(x_{i,A}u_{i,B} + x_{i,B}u_{i,A}), \tag{39}$$

$$v_{AB} = x_{i,A}u_{i,B} + x_{i,B}u_{i,A}, \tag{40}$$

$$v_{ABC} = x_{i,A,C}u_{i,B} + x_{i,B}u_{i,A,C}, \tag{41}$$

then we have

$$E_{AB}^* = E_{AB} + e_{AB}, \tag{42}$$

$$E_{A:B}^* = E_{A:B} + v_{AB}, \tag{43}$$

$$E_{A:BC}^* = E_{A:BC} + v_{ABC}. \tag{44}$$

Substituting from eqns (42)–(44) into eqns (29)–(33), we find that

$$P_{AB}^* = P_{AB} + J_{AB}^{(1)}, \tag{45}$$

$$S_{BA}^* = S_{BA} + J_{BA}^{(2)}, \tag{46}$$

$$T_{KBA}^* = T_{KBA} + J_{BAK}^{(3)}, \tag{47}$$

$$\rho_0\eta^* = \rho_0\eta + J^{(4)}, \tag{48}$$

$$Q_K^* - Q_K + \tau(\dot{Q}_K^* - \dot{Q}_K) = -a_K\theta - k_{KL}\theta_{,L} - h_{KMNe_{MN}} - h'_{KMNV_{MN}} - h_{KMNLV_{MNL}}, \tag{49}$$

where

$$J_{AB}^{(1)} = -B_{AB}\theta + D_{ABCD}v_{CD} + D_{ABCDK}v_{CDK} + C_{ABCD}e_{CD}, \tag{50}$$

$$J_{BA}^{(2)} = -B_{A:B}\theta + D_{ABCD}e_{CD} + G_{ABCDK}v_{CDK} + C'_{ABCD}v_{CD}, \tag{51}$$

$$J_{BAK}^{(3)} = -B_{A:BK}\theta + D_{CDABK}e_{CD} + G_{CDABK}v_{C:D} + C_{ABKLMN}v_{LMN}, \tag{52}$$

$$J^{(4)} = d\theta + B_{AB}e_{AB} + B_{A:B}v_{AB} + B_{A:BK}v_{ABK}. \tag{53}$$

From eqns (7), (8), (21), (22) and (45)–(47), we get

$$T_{Bi}^* = T_{Bi} + u_{i,A}P_{AB} + u_{iC}S_{BC} + u_{iC,K}T_{KBC} + x_{i,A}J_{AB}^{(1)} + x_{iC}J_{BC}^{(2)} + x_{iC,K}J_{CBK}^{(3)}, \tag{54}$$

$$S_{KiB}^* = S_{KiB} + u_{i,A}T_{KAB} + x_{i,A}J_{BAK}^{(3)}. \tag{55}$$

If we denote

$$\pi_{Bi} = T_{Bi}^* - T_{Bi}, \quad N_{AB} = S_{AB}^* - S_{AB}, \quad M_{KKM} = S_{KKM}^* - S_{KKM},$$

$$\Phi_K = Q_K^* - Q_K, \quad \gamma = \eta^* - \eta,$$

then we have

$$\pi_{Bi} = u_{i,A} P_{AB} + u_{iC} S_{BC} + u_{iC,K} T_{KBC} + x_{i,A} J_{AB}^{(1)} + x_{iC} J_{BC}^{(2)} + x_{iC,K} J_{CBK}^{(3)}, \quad (56)$$

$$N_{BA} = J_{BA}^{(2)}, \quad (57)$$

$$M_{KIB} = u_{i,A} T_{KAB} + x_{i,A} J_{BAK}^{(3)}, \quad (58)$$

$$\rho_0 \gamma = J^{(4)}, \quad (59)$$

$$\Phi_K + \tau \dot{\Phi}_K = -a_K \theta - k_{KL} \theta_{,L} - h_{KMN} e_{MN} - h'_{KMN} v_{MN} - h_{KMNL} v_{MNL}. \quad (60)$$

Subtracting eqns (1), (2) and (3) from eqns (15), (16) and (17), respectively, we find the incremental equations to be

$$\pi_{Ki,K} + \rho_0 F_i = \rho_0 \ddot{u}_i, \quad (61)$$

$$M_{MIK,M} - x_{i,A} N_{AK} + \rho_0 F_{iK} = \rho_0 Y_{MK} \ddot{u}_{iM}, \quad (62)$$

$$\rho_0 T \dot{\gamma} + \rho_0 \theta \dot{\eta} = -\Phi_{K,K} + \rho_0 S, \quad (63)$$

where

$$F_i = f_i^* - f_i, \quad F_{iK} = f_{iK}^* - f_{iK} \quad \text{and} \quad S = s^* - s. \quad (64)$$

4. LINEAR GENERALIZED THEORY OF PRESTRESSED DIPOLAR THERMOELASTICITY

In this section, we consider the special case when the primary state Ω of the body is identical with that of the initial body Ω_0 so that $x_1 = X_1$, $x_2 = X_2$, $x_3 = X_3$, and we suppose that Ω_0 is subjected to an initial stress and an initial heat flux caused by the nonuniform initial temperature T_0 . Due to the action of external loadings and heat sources, the body Ω_0 undergoes a deformation. There arise displacements $u_i = \varepsilon u'_i$, $u_{iK} = \varepsilon u'_{iK}$, and temperature acquires an increment $\theta = \varepsilon \theta'$. Here we systematically neglect all powers of ε above the first, except in the free energy function Ψ where we retain quadratic terms in ε .

The work of previous section can be applied immediately to this special case and yields a generalized linear thermoelasticity theory in the presence of initial stress and heat flux. In this case, we have

$$x_{i,A} = \delta_{iA}, \quad J = 1, \quad T = T_0, \quad \dot{\eta} = 0, \\ E_{AB} = 0, \quad E_{A:B} = 0, \quad E_{A:BK} = 0.$$

Obviously, all coefficients defined in the previous section are now evaluated at $E_{KL} = 0$, $E_{A:B} = 0$, $E_{A:BK} = 0$, and $T = T_0$. From eqns (39)–(41), we find that

$$e_{AB} = \frac{1}{2}(u_{A,B} + u_{B,A}), \quad (65)$$

$$v_{AB} = u_{AB} + X_{CA} u_{C,B}, \quad (66)$$

$$v_{ABC} = X_{KA,C} u_{K,B} + u_{BA,C}. \quad (67)$$

The governing equations (61)–(63) become

$$\pi_{KA,K} + \rho_o F_A = \rho_o \ddot{u}_A, \quad (68)$$

$$M_{MAK,M} - N_{AK} + \rho_o F_{AK} = \rho_o Y_{MK} \ddot{u}_{AM}, \quad (69)$$

$$\rho_o T_o \dot{\gamma} = -\Phi_{K,K} + \rho_o S. \quad (70)$$

The constitutive relations (56)–(60) reduce to

$$\pi_{BK} = u_{K,A} P_{AB} + u_{KC} S_{BC} + u_{KC,N} T_{NBC} + J_{KB}^{(1)} + X_{KC} J_{BC}^{(2)} + X_{KC,N} J_{CBN}^{(3)}, \quad (71)$$

$$N_{BA} = J_{BA}^{(2)}, \quad (72)$$

$$M_{KNB} = u_{N,A} T_{KAB} + J_{BNK}^{(3)}, \quad (73)$$

$$\rho_o \dot{\gamma} = J^{(4)}, \quad (74)$$

$$\Phi_K + \tau \dot{\Phi}_K = -a_K \theta - k_{KL} \theta_{,L} - h_{KMN} e_{MN} - h'_{KMN} v_{MN} - h_{KMNL} v_{MNL}. \quad (75)$$

The functions P_{AB} , S_{BC} and T_{NBC} characterize the initial stress and they can be arbitrary functions apart from satisfying the energy equation for the static case and the condition that Ω_o is in equilibrium. The presence of the initial heat flux vector is determined by the nonuniformity of the initial temperature T_o . This fact implies the appearance of the coefficients a_K , h_{KMN} , h'_{KMN} and h_{KMNL} in the expression for Φ_A . If T_o is constant, then $Q_A = 0$, $a_K = 0$, $h_{KMN} = 0$, $h'_{KMN} = 0$ and $h_{KMNL} = 0$.

Substituting from eqns (74) and (75) into the heat conduction equation (70), we find that

$$\begin{aligned} T_o \left(1 + \tau \frac{\partial}{\partial t} \right) (d\theta + B_{AB} \dot{e}_{AB} + B_{A:B} \dot{v}_{AB} + B_{A:BK} \dot{v}_{ABK}) \\ = (a_K \theta + k_{KL} \theta_{,L} + h_{KMN} e_{MN} + h'_{KMN} v_{MN} + h_{KMNL} v_{MNL})_{,K} + \rho_o \left(1 + \tau \frac{\partial}{\partial t} \right) S, \end{aligned} \quad (76)$$

which may be a hyperbolic equation if the coefficients have proper signs. As a consequence, heat will propagate at a finite speed and the paradox of infinite speed is thus circumvented.

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